Cybernetics and Human Knowing. Vol. 24 (2017), nos. 3-4, pp. 5-15

Foreword: Laws of Form

Louis H. Kauffman

> The theme of this book is that a universe comes into being when a space is severed or taken apart. The skin of a living organism cuts off an outside from an inside. So does the circumference of a circle in the plane. By tracing the way we represent such a severance, we can begin to reconstruct, with an accuracy and coverage that appear almost uncanny, the basic forms underlying linguistic, mathematical, physical, and biological science, and can begin to see how the familiar laws of our own experience follow inexorably from the original act of severance.—George Spencer-Brown, 1969, p. v of *Laws of Form,* a note on the mathematical approach

This special issue devoted to George Spencer-Brown (April 2, 1923 – August 25, 2016) and his seminal work *Laws of Form* (Spencer-Brown, 1969). Spencer-Brown has been a key figure in the foundations of second-order cybernetics initially through the work of Humberto Maturana, Francisco Varela and Louis H. Kauffman (Varela, Maturana, & Uribe, 1974; Varela, 1979; Kauffman & Varela, 1980). Spencer-Brown's work has been the subject of much cybernetic musing by many workers including Ranulph Glanville, Nicolas Luhmann and many others. His work strikes a deep chord for all scholars by pointing to the fundamental role of the act of making a distinction or imagining a distinction in the formation of knowledge of any kind. As the reader will see, this work of Spencer-Brown comes as a particular culmination of many aspects of epistemology. In his review of *Laws of Form* in the *Whole Earth Catalog*, Heinz von Foerster wrote:

> The laws of form have finally been written! With a "Spencer-Brown" transistorized power razor (a Twentieth Century model of Occam's razor) G. Spencer-Brown cuts smoothly through two millennia of growth of the most prolific and persistent of semantic weeds, presenting us with his superbly written *Laws of Form*. This Herculean task which now, in retrospect, is of profound simplicity rests on his discovery of the form of laws. Laws are not descriptions, they are commands, injunctions: "Do!" Thus the first constructive proposition in this book (page 3) is the injunction: "Draw a distinction!" an exhortation to perform the primordial creative act. (Von Foerster, 1971, p. 12)

We dedicate this special issue to (the memory of) G. Spencer-Brown, and to the continuing understanding of his basic insight.

Here follows a description of the form of this special issue and a summary of some of its ideas and directions. Most of the articles are examinations of ideas related to or emanating from Spencer-Brown's work. The article by Graham Ellsbury has a special historical value in that it is a remarkable reminiscence of a long friendship and working relationship with G. Spencer-Brown. that casts light on the personal motivations for LoF. I shall describe each article briefly, but first I would like to say a few words about Spencer-Brown's work. In this preface I shall use laws of form or the notation LoF to refer to the ideas and content of the work *Laws of Form*.

In *Laws of Form*, Spencer-Brown makes a calculus of extraordinary simplicity. This *calculus of indications* is a combinatorial mathematical system (an *arithmetic* in Spencer-Brown's terms) that is generated by a single sign, $\urcorner$, called the *mark*. The mark is drawn in the notational plane and is regarded as having an inside and an outside. One can regard the mark as an abbreviated box $\square$ and it is seen to make a distinction in the plane as that box, delineating an inside and an outside. Thus $\overline{\urcorner}|$ connotes a mark within a mark just as $\boxed{\square}$ connotes a box within a box, and $\urcorner\urcorner$ connotes a mark next to a mark just as $\square\square$ connotes a box next to a box. In order to use this notation the reader must be able to see the mark as making a distinction in the plane. In most mathematics it is assumed that the reader is familiar with all the usual typographical and geometric conventions. Most mathematical texts do not discuss the foundational distinctions that are part of their own typography. In LoF one finds one is thrown into a long contemplation of exactly this aspect of foundations. The reader or user of the system must be able to make these distinctions in order to comprehend the system. A mathematical system is built from notations that connote distinctions.

Spencer-Brown does not start with the notation for the mark. The first lines of the book are:

> We take as given the idea of distinction and the idea of indication, and that one cannot make an indication without drawing a distinction. We take, therefore, the form of distinction for the form. (Spencer-Brown, 1969, p. 1)

Laws of form is based on the idea of distinction. A circular solution to the problem of form is presented in the second sentence of the book. We take the form of distinction for the form. This gives the reader a project—to examine the notion, the form of distinction. What is the form of distinction? A distinction can be made. There must be two parts that are distinct. Let one of them be marked, the other unmarked. We already have the idea of distinction and many examples of distinctions that we are willing to bring forth.

I could close the book, examine all sorts of distinctions, myself/the world, light/dark, marked/unmarked, male/female and ask what is the form of distinction. Or I can continue reading the book. If I would continue reading, I am confronted with the next lines.

> **Definition**
> *Distinction is perfect continence.*
> That is to say, a distinction is drawn by arranging a boundary with separate sides so that a point on one side cannot reach the other side without crossing the boundary. (Spencer-Brown, 1969, p. 1)

Continence is meant in the sense of containment, a perfect containment. *Definition* means distinction. A definition arranges a boundary so that one can distinguish that a something does or does not satisfy the definition. If definition means distinction, then distinction has not been defined in terms of something else. These words are synonyms for distinction. To draw a boundary is an example of a distinction. We can illustrate distinctions by making marks and boxes. *Illustration* is a form of indication. We cannot make an indication without drawing a distinction. When we call (as is speak or call out) the name of a distinction or a side of a distinction, we are making a distinction. When we cross the boundary of a distinction we are making a distinction. All the words in the language each make distinctions in their own way. As Spencer-Brown says in his preface:

> The act is itself already remembered, even if unconsciously, as our first attempt to distinguish different things in a world where, in the first place, the boundaries can be drawn anywhere we please. At this stage, the universe cannot be distinguished from how we act upon it, and the world may seem like shifting sand beneath our feet. (Spencer-Brown, 1969, p. v)

One's initial encounter with LoF is an entry into this confusion where all distinctions, thought and language are seen to be aspects of each other, all aspects of the form of distinction. We know that language, our language, describes itself and can comment on itself. But when we go to the roots of language, there is only the possibility of a distinction. There seems to be nothing to establish and no place to begin. And yet, Spencer-Brown begins. The condensations of distinctions are brought from all of language to two motifs that are called *calling* and *crossing*. Calling, in this specific sense, means the calling of a name, as one might call out "It is morning, morning.". In this sentence I have called the name of the time of day (morning) twice, perhaps for emphasis, but the indication of morning requires only one speaking or calling of that name. Thus the law of calling below, states that the value of a call made again is the value of a call. This is the same as saying that if I should call your name twice in succession, the value of that indication is the same as if I were to call your name once. For the law of crossing, the reader can visualize a circle on the ground in a field, making a distinction between inside and outside. The operation of crossing the boundary will take someone standing inside the circle to the outside or someone standing on the outside to the inside. If one crosses twice, then the result is the same as that of not having crossed at all. This is, of course, the simplest scenario that one can think about. We do not assume that the observer is standing on the boundary, or that we are in the situation of extra distinctions where the inside has changed in the time it takes to make two successive crossings.

- The law of calling: The value of a call made again is the value of the call.
- The law of crossing: The value of a crossing made again is not the value of the crossing.

To call is to name and the law of calling says that calling a name twice has the same value as calling it once. To cross is to cross the boundary of a distinction. Since we consider only one distinction, to cross again is the same as not to cross. What is not marked is unmarked. What is not not marked is what is not unmarked, and that is marked.

The two laws are imaged in equalities in the calculus of indications:

law of calling. $\overline{}|\,\overline{}| = \overline{}|$

law of crossing. $\overline{\overline{}|}| =$

In order for these representations to make sense, one interprets the outside of an empty mark as marked. Then an extra mark on the outside constitutes calling the name again and the two calls have the same value as a single call. One also interprets the mark as crossing from the state indicated on its inside. Thus $\overline{\overline{}|}|$ indicates crossing from the marked state (which is inside the outer mark) and so indicates the unmarked state.

This calculus of indications can be regarded as describing operations relative to a single distinction. The mark can be an instruction to cross the boundary of that distinction. The mark is the name of the outside of that distinction. Operation and name are interchangeable in the form. Furthermore, since the mark itself makes a distinction (i.e., we make the distinction of the mark and the mark is an indication of that distinction) we can regard the primary distinction of the calculus of indications to be its own mark. The calculus refers to itself. The calculus is, with our help, speaking about itself. At the end of the book Spencer-Brown says:

> We see now that the first distinction, the mark, and the observer are not only interchangeable, but, in the form, identical. (Spencer-Brown, 1969, p. 76)

We as observers need a remarkable panoply of distinctions to discuss the simplicity of one distinction and the mathematics, the calculus of indications, that comes in its wake. After arithmetic there is algebra and after algebra there is the possibility of the algebra acting on itself and producing recursion, self-reference, imaginary values. After that there is a return to the beginning. The last injunction of the book is to return to the beginning of the book. The injunction to understand that "the form of distinction is the form" is to go back to the beginning, prior to the formalism, prior to the fixed assumptions of all kinds, and create it again. In the doing arises understanding.

Laws of Form is related to many aspects of mathematics and it is, for many, fundamental to cybernetics. Those of us writing this volume feel that Spencer-Brown's ideas and his concise formulation of them in this book should be more widely known and understood. We have not taken up or discussed any of the criticism that *Laws of Form* has received, nor have we adequately discussed the mathematical and philosophical background of the book. It is worth mentioning that in the background is the whole twentieth century development of symbolic logic starting in the nineteenth century with Boole, DeMorgan, Lewis Caroll (Charles Dodgson), Cantor, Frege, Peano, Charles Sanders Peirce, Russell and Whitehead and continuing with Brouwer, Hilbert, Ludwig Wittgenstein, Kurt Goedel, Paul Cohen and onward into category theory with Samuel Eilenberg, Saunders MacLane and William Lawvere. It is in category theory and topology where there is a return to diagrammatic geometry going beyond typographical logic.

I mention this panoply of names without formal reference since the reader can find them well enough in his own bibliographic searches. I mention the paper of mine on the mathematics of Charles Sanders Peirce (Kauffman, 2001) for its comparison of the existential graphs of Peirce with the notation and structure of *Laws of Form*. In that paper, I show that Peirce and Spencer-Brown come remarkably close to each other (with Peirce long preceding Spencer-Brown) and that Spencer-Brown's natural use of the unmarked state is related to Peirce's plane of assertion. These explorations of diagrammatic epistemology are, in my opinion of very great importance and they distinguish both Peirce and Spencer-Brown from the rest of the logic tradition that works almost exclusively with standard typography. It is with Goedel that the standardization of the typography of a formal system comes to a head, with the Goedel numbering coding the elements of the formal system as specific natural numbers. With the possibility of such coding, formal systems that also encapsulate number theory are shown by Goedel to be incomplete and unable to prove their own consistency (if they are given to be consistent). This occurs by the use of indirect self-reference mediated by the Goedelian coding. One way out of this trap is to allow formal systems whose base signology is always expanding and not fixed. Then there is no full Goedel coding possible for such expanding systems. This remark is meant to point to the need for a flexible typography at the base of mathematics. In our point of view this expands to the desire for a flexible concept of mathematics as arising from the possibility of distinctions, and not from some particular systems of axioms. In this way the book *Laws of Form* becomes a nexus for the discussion of the formal nature of the foundations of mathematics. It is a very special situation that such a simple system and concise text can form the center of a discussion of this magnitude. The famous mathematician John H. Conway has quipped that *Laws of Form* is "beautifully written and content free." Just so, it is we who create the content in the course of imagining the distinctions that become our mathematics or our science or our cybernetics. Spencer-Brown has discovered what may be the simplest non-trivial formal system.

I hope that this sketch of laws of form whets the appetites of readers who have not yet encountered it. Readers familiar with second-order cybernetics can see from this introduction the deep link between considerations of including the observer and the very roots of language and thought as articulated through Spencer-Brown. Let us turn now to a guide to the papers in this issue.

Discussion of Articles

In "A Calculus of Negation in Communication" Dirk Baecker compares the theory of information and communication due to Claude Shannon with the structure and notions of information in Spencer-Brown's *Laws of Form*. Shannon's approach to information is structural for the sake of engineering practice and related to uncertainty in the sense that Shannon would understand that a message contains information for an observer to the extent that the reception of the message can make a difference (as with Bateson, for example) for that observer. The pivot is difference/distinction and when we make that turn we find ourselves in the domain of Spencer-Brown. For Spencer-Brown information is in-formation. The form re-enters its own indicational space. Baecker discusses these relationships and points out how "…it is Shannon's sense for the necessity to conceive of the outside of the message, be it well-defined as in engineering or contextualized as in culture, that, like a hidden challenge, informs any kind of 'theoretical' thinking in the 20th century. It is only Spencer-Brown's form which actually allows us to realize that outside." There are many dimensions to Baecker's essay. The comparison between Spencer-Brown and Shannon is worthwhile and worth following further. They both worked with the relationship of circuits and Boolean logic. Shannon used switching circuits while Spencer-Brown used logic gates generated by distinction operators. There is much more to be explored in these domains. I should point out that the essay by myself and Christina Weiss (Kauffman & Weiss, 2001) on similar relationships between circuits and the conceptual notation of Gottlob Frege. There remains an undiscovered country where the Boolean and the digital is suffused with meaning as in *Laws of Form*.

William Bricken's article "Distinction Is Sufficient: Iconic and Symbolic Perspectives on *Laws of Form*" is a lucid discussion of laws of form coupled with a creative panoply of diagrammatic and geometric uses of formalisms related to the Spencer-Brown calculus. As in Hellerstein's paper, Bricken uses a parenthesis as in < > for the Spencer-Brown mark, and he continues in this line and then makes a transition to network models, computation and discussions of iconics. Bricken's constructions may lead to new forms of computation and they constitute a significant exploration of typographical foundations as we have discussed above.

Art Collings's paper, "The Brown-4 Indicational Calculus," is a concise exposition of his significant generalization of the Spencer-Brown mark to a four-fold operator, the algebraic theorems and completeness theorems that ensue from this. The paper ends with an explanation of a relationship (due to Collings and Kauffman) of Collings formalism with the complex numbers.

Graham Ellsbury's article, "George Spencer-Brown as I Knew Him—A Brief Personal Memoir," is a remarkable reminiscence of his long relationship with Spencer-Brown. The paper includes quite a bit of mathematics in the form of its story, and provides a wider view of LoF by including its author in the discussion. Ellsbury makes a point about the interpretation of LoF for logic that has surely captivated many readers of Spencer-Brown. In formal logic we learn that "P implies Q" is to be represented by $\neg P \vee Q$ where $\neg$ stands for "not" and $\vee$ for "or." This curious logical contraption should have a more fundamental mode of expression, and indeed it does in LoF with $\overline{P|}\,Q$ taking the role of "P implies Q." In his essay, Ellsbury expresses very well what many have exclaimed on first seeing this convergence into the form of distinction: "As I read through *Laws of Form*, I exclaimed: 'This is it! Spencer-Brown's boundary is the primitive sign of logic!'"

Jack Engstrom's paper, "System E — A New Language that Reveals New Distinctions in Laws of Form's Notational Space," is an exposition of his iconic notations for subtleties in the form. In particular he articulates ways to think about the *void lines* that are implicit throughout the Spencer-Brown notation. The simplest example here is the mark itself in the form of the right-angle bracket ⏋. The mark is intended to make a distinction between an outside and an inside and it is implicit in the use of this notation that the letter A is inside $\overline{A}|$ and outside the mark in the expression ⏋A. One way to say this aspect of the mark's distinction is to say that the mark ⏋ is an abbreviation for the box □. Having said this we see that the box is a composite of the mark ⏋ and two more lines in the form └. Thus one can regard the mark as consisting in four lines (line segments) two of which are void. Engstrom introduces the notion of void lines and makes an analysis of their role in the arithmetic and algebra of laws of form. This articulation of void lines is in fact paralleled by Spencer-Brown's own work in his explorations of the map color theorem (Spencer-Brown, 1980), and Engstrom's work helps set the stage for a basis for this mathematics.

Nathaniel Hellerstein's paper, "Diamond bracket forms and how to count to two," is a concise exposition of his work in generalizing the algebra and logic of Spencer-Brown. He uses a typographical notation for the Spencer-Brown mark in the form < > so that the laws of calling and crossing become < > < > = < > and << >> =   . This representation can be easily produced with standard typography. That is its advantage. The conceptual disadvantage is that it breaks the Spencer-Brown mark into a combination of two distinctions, the left bracket and the right bracket. The advantage of this disadvantage is that one can examine the relationships of the brackets with the way parenthesization creates a framework of distinctions in ordinary typography. He also introduces the symbols 0 for the unmarked state and 1 for the marked state so that we have 1 = < > and 0 = << >>. He then introduces an extended arithmetic called by him "Diamond" with two new values 6 and 9 so that <6> = 6 and <9> = 9. These have been originally introduced by Kauffman (1978), Kauffman and Varela (1980)] as i and j so that <i>= i and <j> = j and ij = <>. The wave-form arithmetic of Kauffman and Varela has been useful for thinking about self-reference and in the hands of Hellerstein the four values become a way of thinking paradoxically about all subjects. Diamond is a four-valued arithmetic with its own rules that can be articulated in the context of the Spencer-Brown notations. This was the original idea of Kauffman and Varela to extend laws of form to include these imaginary values so that one could explore the possibilities for reasoning and construction that are implicit in values beyond the True and the False. These matters are still under exploration.

Self-reference and fixed points are inherent in Diamond and Hellerstein goes on to prove theorems about fixed points and to analyze modulators (special circuits related to Chapter 11 of *Laws of Form*) in terms of Diamond and global recursion. It remains to be seen what will be the practical applications of such explorations. This remark applies equally to the work of Collings and the work of Kauffman in this volume. The paper as a whole is of interest not just for its contents, but as an exercise in the limits of using typographical forms to express matters that involve a second dimension.

Louis Kauffman's article, "Imaginary values," is about imaginary and self-referential values in laws of form. The paper begins by considering the iconic role of boundaries in both LoF and in Venn diagrams. From the considerations of boundaries and diagrams, he describes how different calculi arise that are relatives to the calculus of indications. This includes the calculus of idempositions that Spencer-Brown uses in his work on the map coloring problem. From consideration of boundaries and diagrams he shows how non-standard logics such as Heyting algebras and co-Heyting algebras arise. The point is that it is natural to bring the boundary of a distinction into view and see how, in the simplest possible ways, the boundary is related to the original distinction. For example, the reader has surely used Venn diagrams in learning logic and set theory. If we take the boundaries of the regions in a Venn diagram as part of the mathematical structure, then we find that the logic of the diagrams is naturally interpreted in terms of Heyting and co-Heyting algebras. These algebras have long been part of the foundations of so-called intuitionistic logic and have many interpretations more complex then the properties of a simple boundary. In this paper we show how such ideas and their indications arise from very simple forms. The paper then uses minimal examples of such algebras to give models that satisfy some but not all of the axioms for the Spencer-Brown primary algebra. This is an example of using imaginary Boolean values. In particular we show that reflection is not a consequence of generation and integration. The point of such constructions is that imaginary Boolean values (Spencer-Brown's term) can be used to prove a theorem that would be difficult or impossible using just Boolean values. The paper ends with a discussion of the complex numbers in relation to LoF, and with a discussion of fractals in relation to LoF. Fractals are a very palpable way to explore recursion and to see how, by taking infinite constructions, a form can enter its own indicational space. This use of fractals then recalls our fundamental question to understand how we cognitively enter our own indicational space. We wish to understand the self-referential condition of human observing. By entering into the games of these formal systems, we obtain experience that helps in this quest. Working with formal systems and the circularity of designing and recursion is one way to explore the loop of the self and other, the dialectic of human meaning. There are others. Some would not take these particular formal journeys. We only make invitations. Some may wish to examine basic questions without the guide or crutch of formalism. The theme of imaginary values in this paper is that mathematical constructions of all kinds are themselves imaginary extensions of the power of our reason.

Martin Rathgeb's article, "Re-reading the *Laws of Form* as a language in change," is about the linguistic structure of *Laws of Form* and in this framework carefully walks through the multiple points of view and shifts that occur as one goes from concepts of distinction to the specifics of an arithmetic that utilizes the unmarked state and the unwritten cross, to the referential structure of the primary algebra and to the transition to forms that re-enter their own indicational space. This movement of language is subjected to analysis in relation to logical power, expressive power, methodical power, integrative power, explanatory power and metaphorical power in relation to the work of Ladislav Kvasz.

Christina Weiss's article, "Towards a phenomenology of schematization," is about the philosophical and phenomenological structure of *Laws of Form*. She grapples with the apparent problems of circularity of definition. How can one define a distinction (even as perfect continence) when any definition is surely a form of distinction itself? These problems and related questions about particularity and universality are part and parcel of the act of reading Spencer-Brown and they are deserving of philosophical reflection.

Final Remark: This collection of papers and commentary covers much ground related to LoF. In this remark, I indicate some other directions that could be followed and that we hope to follow in another Special Issue devoted to LoF. First of all, the present collection does not discuss the relationship of laws of form with logic. It is worth filling in that omission here with a brief translation. We can take the mark $\overline{}|$ to stand for True and the unmarked state or its representative $\overline{\overline{}|}|$ to stand for False. Then one observes that juxtaposition of forms as in AB represents $A \vee B$ (A or B) since True $\vee$ True = True corresponds to $\overline{}|\overline{}| = \overline{}|$, the law of calling. Furthermore $\neg$A (not A) is represented by $\overline{A}|$ and the law of double negation is fueled by the law of crossing $\overline{\overline{}|}| =$. With this, we have that implication $A \supset B$ is represented by $\overline{A}|B$. The reader can consult Appendix 2 in *Laws of Form* for more about this direction.

Many of the papers in this special issue discuss boundaries in relation to laws of form. Spencer-Brown's work on the coloring of maps and the four color theorem is directly related to a calculus that uses two colors of curves in the plane that interact with one another to make expressions called formations. This occurs in a calculus with two primary marks and the entire structure of the map coloring problem is translated into a problem about this new calculus. The source for this aspect of Spencer-Brown's work is his paper "Cast and Formation Properties of Maps" (Spencer-Brown, 1980) and it is partially reprinted in the latest edition of *Laws of Form*. His insightful work in number theory related to the Riemann hypothesis is also included in this edition.

Imaginary values have been touched upon in the articles in this volume, but a key use of imaginary values is seen in *Laws of Form*, chapter 11 in the action of the modulator circuits that will be a subject of further work. In examining the action of the modulators, Spencer-Brown notes that certain values occur just in time to make the circuit work as it is desired. These forms of distinctions in time are called imaginary values by him, and we, in our rush to make sense of the idea of imaginary values in logic and mathematics, have not yet captured the subtlety of this transition. More work needs to be done in this domain.

Finally, there are many relationships of laws of form with philosophical and spiritual ideas, from the very beginning of the book where one starts in a realm prior to language, to the many notes to the chapters. I will end this preface with two quotes from those notes:

> It is, I am afraid, the intellectual block which most of us come up against at the points where, to experience the world clearly, we must abandon existence to truth, truth to indication, indication to form, and form to void, that has held up the development of logic and its mathematics. (Spencer-Brown, 1969, p. 101)

> Thus we cannot escape the fact that the world is constructed in order (and thus in such a way as to be able) to see itself. (Spencer-Brown, 1969, p. 105)

Regular Features

Column

Louis Kauffman's Virtual Logic column is a fictional dialogue between the sentient text strings Cookie and Parabel, who talk about laws of form and Spencer-Brown from the point of view of their unique epistemology, always very near the void. The main theme of this column is how levels of meaning arise in relation to simple formalism.

> **Cookie:** What does he mean by levels of meaning? We are just typographical strings.
> **Parabel:** We seem to make commentary on each other's string. How do we do that?
> **Cookie:** Oh Parabel, how could a string of letters comment on anything?

Featured Artist

"The form we take to exist arises from framing nothing" (Spencer-Brown, 1969, p. 105). A form which frames nothing was found in the shape of 5-dimensional space by this issue's featured artist: the Vienna based geometer, art researcher, philosopher and science communicator Renate C.-Z.-Quehenberger. Her ontological research, which combines philosophy with geometry is inspired by Gregory Bateson and Heinz von Förster's quest for "the pattern which connects."[1]

Her associated contributions are in both art and science contexts: Her visualizations include 3D animated geometry films, installations and objects exhibited in art spaces and museums. Quehenberger's investigations have led her to the development of a new 3D animated geometry for higher mathematics for the visualization of quantum phenomena (as executed in her art research project "Quantum Cinema a digital Vision."[2]

Her work is associated with—among others—Sir Roger Penrose's kites and darts,[3] Poincaré's homology sphere,[4] and the Planck scale.[5]

Quehenberger's artwork presented here connects the ancient theory of everything with 20th century physics by following the truly Platonic aim for visualizing the ideal world by means of digital arts. Plato's triangles cradled by the wet-nurse-of- becoming becomes alive in the higher dimensional realms of quantum physics and Spencer-Brown's circles gain a new relevance.

1. See her thesis, *On the Hermeneutics of the Penrose Patterns*, where she hypothesizes that the geometry of 5-dimensional space is the ontologic basis that existence precedes.
2. Quantum Cinema - a digital vision, Art research project with Peter Weibel including the quantum physicist Helmut Rauch, the philosopher Elisabeth von Samsonow and the geometer Helmut Stachel, funded by the Austrian Science Fund, FWF No. AR 35-G21
3. (2014). A newly discovered Heptahedron named Epitahedron. *Symmetry: Culture and Science, 25(*3),177–192
4. Visualizing Poincaré's dream. Paper presented at SEOUL ICM 2014, International Congress of Mathematicians, Seoul. August 18, 2014
5. A Proposal for a Psi-ontological model based on 5-dimensional Geometry, QCQMB Workshop: Quantum Contextuality in Quantum Mechanics and Beyond, Prague 2017

References for the Discussion of Articles

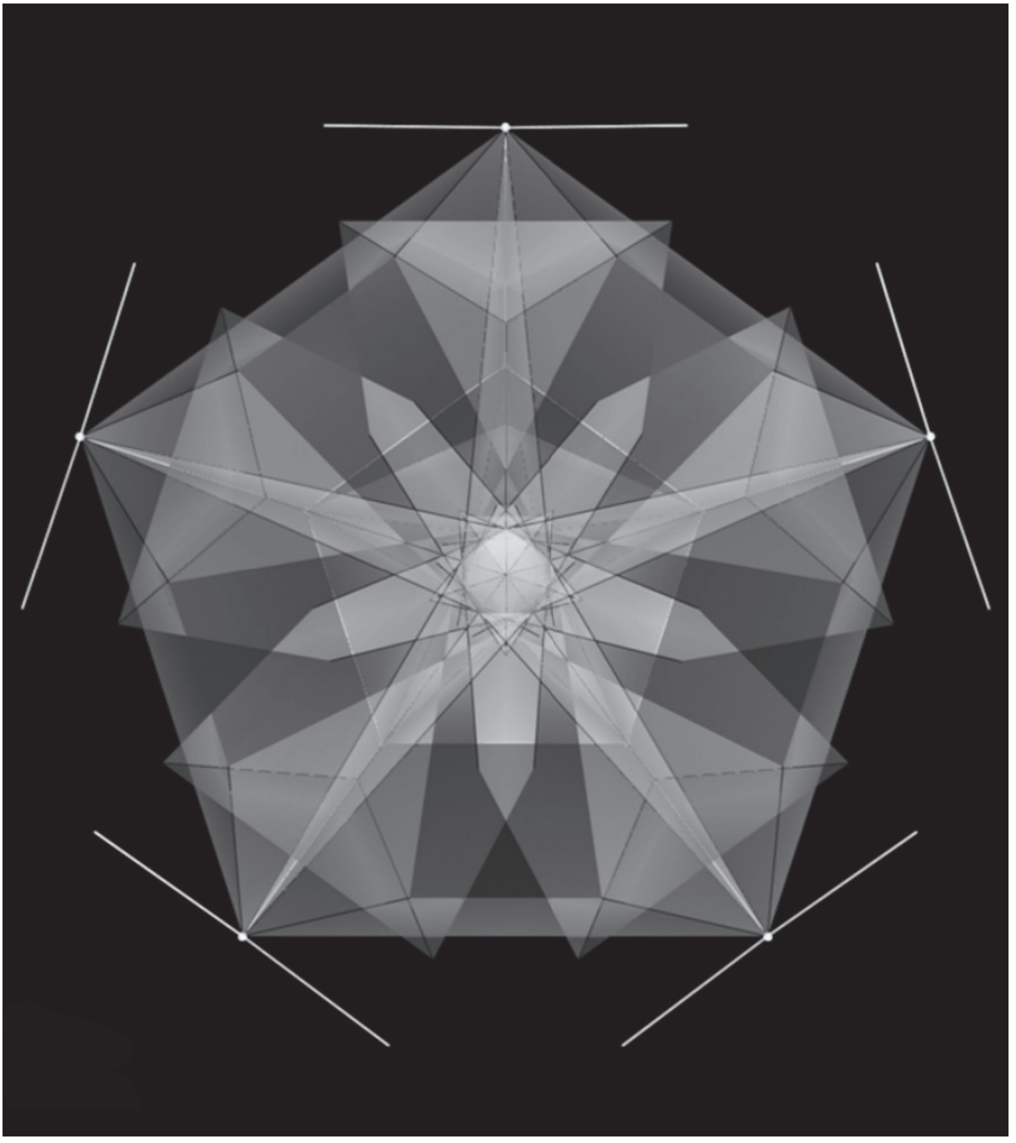
Kauffman, L. H. (1978b). DeMorgan Algebras—Completeness and recursion. In *Proceedings of the Eighth International Conference on multiple valued logic* (pp. 82–86). New York: IEEE Computer Society Press.
Kauffman, L. H., & Varela, F. (1980). Form dynamics. *Journal of Social and Biological Structures*, *3*, 171–206. Retrieved November 24, 2017 from: http://cepa.info/1841
Kauffman, L. H. (2001). The mathematics of Charles Sanders Peirce. *Cybernetics and Human Knowing, 8*(1-2), 79–110.
Kauffman, L. H., & Weiss, C. (2001). Virtual Logic—The key to Frege. *Cybernetics and Human Knowing, 8*(4), 75–86.
Spencer-Brown, G. (1969). *Laws of form*. London: George Allen and Unwin Ltd.
Varela, F. J. (1979). *Principles of biological autonomy*. New York: North Holland.
Varela, F. J., Maturana H. R., & Uribe R. (1974). Autopoiesis: The organization of living systems, its characterization and a model. *Biosystems, 5*(4), 187–196. Retrieved November 24, 2017 from: http://cepa.info/546
Von Foerster, H. (1971). Review of *Laws of Form* by G. Spencer-Brown. In S. Brand (Ed.), *The Last Whole Earth Catalogue* (p. 12). Harmondsworth, UK: Penguin Books.

Editor's Remark on the Bibliography

I have included a complete (up to my abilities) bibliography of Spencer-Brown's writings. This includes many different editions of *Laws of Form*. These editions are worth examining separately. Each has its own introductions that are valuable. It would be a good idea to collect them all in one place, but in the publications they are distributed among the editions. The very last edition of the book contains articles on numbers, number theory and the coloring of maps that are highly valuable to examine along with the original text of *Laws of Form*.—Lou Kauffman.

Bibliography for G. Spencer-Brown

Keys, J. (1970). *Twenty-three degrees of paradise*. Cambridge, UK: Cat Books. (James Keys is a pseudonym for G. Spencer-Brown)
Keys, J. (1971). *Only two can play this game*. Cambridge, UK: Cat Books. (James Keys is a pseudonym for G. Spencer-Brown)
Keys, J. (1972). *Only two can play this game*. New York: Julian Press. (James Keys is a pseudonym for G. Spencer-Brown)
Keys, J. (1974). *Only two can play this game*. New York: Bantam. (James Keys is a pseudonym for G. Spencer-Brown)
Spencer-Brown, G. (1957). *Probability and scientific inference*. London: Longmans, Green & Co.
Spencer-Brown, G. (1961). *Design with the Nor*. Technical Pub. Dept., Mullard Equipment Ltd. Manor Royal, Crawley, Sussex: Mullard Equipment Ltd.
Spencer-Brown, G. (1965). British patent specifications 1006018 Lift Control System and 1006019 Lift Control Circuit Arrangement. Intellectual Property Office, U.K.
Spencer-Brown, G. (1969). *Laws of form*. London: Allen and Unwin.
Spencer-Brown, G. (1972). *Laws of form*. New York: Julian Press.
Spencer-Brown, G. (1973). *Aum conference transcript*, Esalen Conference, March 1973 (unpublished transcript). Retrieved December 19, 2017 from: http://www.uboeschenstein.ch/texte/spencer-brown.html
Spencer-Brown, G. (1979). *Laws of form*. E. P. Dutton, New York.
Spencer-Brown, G. (1994a). *Laws of form*. Portland, OR: Cognizer Connection.
Spencer-Brown, G. (1994b). A Lion's teeth. Soltendieck, Germany: Bohmeier Verlag.
Spencer-Brown, G. (1998). Uncolorable trivalent graphs. *Cybernetics and Systems, 29*(4), 319–344.
Spencer-Brown, G. (2008). *Laws of form*. Soltendieck, Germany: Bohmeier Verlag.
Spencer-Brown, G. (2008). An algebra for the natural numbers. In *Laws of form* (pp. 120–126). Soltendieck, Germany: Bohmeier Verlag. (Originally published in 1961)
Spencer-Brown, G. (2008). Cast and Formation Properties of Maps. Excerpt published in *Laws of form* (pp. 127–179). Soltendieck, Germany: Bohmeier Verlag. (Originally published in 1980; Deposited in the Royal Library, Cambridge, UK)



Quehenberger, R. (2012). *Passage to Level 1, Quantum Cinema*. A digital cut through the epita-dodecahedron. Video animation, still at timecode 00:01:32.