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# The Brown-4 Indicational Calculus

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This paper introduces the Brown-4 indicational calculus, a new four-valued extension to George Spencer-Brown's calculus of indications. The B4 calculus is distinguished by its foundation on a new axiomatic basis, its fidelity to Spencer-Brown's original notation, and because it can be interpreted as a wide range of systems, in particular as a 4-valued logic known as a bilattice. B4 has theoretical and practical significance for cybernetics, due to the intrinsic presence in the calculus of expressions with fixed point solutions.
**Keywords:** laws of form, imaginary Boolean values, fixed points, eigenvalues, cyclic operation, square root of negation, indicational notation, distinction, modal logic, bilattice

## Introduction

This article introduces the Brown-4 indicational calculus, or B4, a new 4-valued extension to the calculus of indications of George Spencer-Brown (Spencer-Brown, 1969), and is inspired by earlier work by Francisco Varela and Louis H. Kauffman. I'm pleased to offer this introduction to the B4 calculus in remembrance of and tribute to Spencer-Brown.

The B4 calculus adds two states (or values) to Spencer-Brown's original calculus, which are directly analogous to the concept of the square roots of negative one. The B4 calculus fully incorporates *i* as a cyclic operator, in contrast to the earlier 4-valued system introduced by Varela and Kauffman, to which B4 is closely related and which also uses imaginary values (see Appendix). We introduce a powerful, previously unknown axiom set that permits the derivation of new logical consequences, and correspondingly a greatly widened range of possible interpretations for the calculus.

The B4 calculus introduces a new token, the reversed mark

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which is recognizably distinct from Spencer-Brown's mark of distinction. The new symbol is linked to the concept of movement in a cycle of four states, such as a clock whose movements consist of quarter hours. The B4 calculus retains a high level of fidelity to the original two-valued calculus, so that every expression using Spencer-Brown's *crossing* operation ┐ remains valid in the new calculus. Readers who are acquainted with laws of form will find themselves on familiar as well as unfamiliar ground.

The paper begins with a detailed description of the extended notation employed in B4, which is based on the concept of a cycle of length four. After these preliminaries, it introduces the revised initials for the new calculus, and then introduces the reader to

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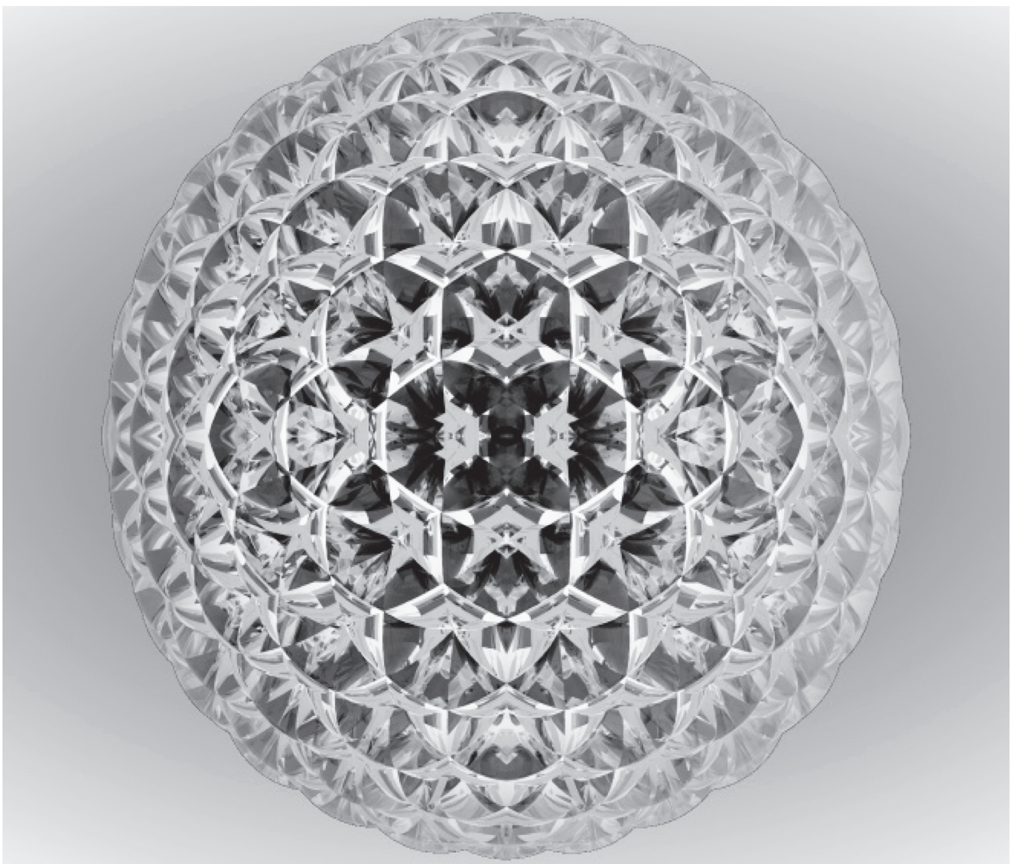
$\sim\Box(Y \supset X)$ $\qquad\qquad$ $X = \Diamond\, Y$

It is very interestingly to note that our modal construction of S5 is equivalent to the conjunction of the two mirror forms $\Box_{GL}$ and $\Box_{GL*}$ together with the variable X:

$$\Box_{S5}\, X = \overline{\overline{\overline{\lvert\overline{X}\ \lvert\overline{\lvert\overline{\lvert\overline{\ }}}}\rvert\ \overline{\lvert\overline{\lvert\overline{\lvert\overline{X}}}\ \lvert\overline{\ }}\rvert}\rvert\ \overline{X}\rvert}\rvert = \overline{\overline{\lvert\overline{X}\ \lvert\overline{\ }}\rvert\ \overline{\lvert\overline{\lvert\overline{\lvert\overline{X}}}\ \lvert\overline{\lvert\overline{\lvert\overline{\ }}}}\rvert\ \overline{X}\rvert}\rvert\,. \qquad \text{(B10)}$$

The study of fixed points in B4 is promising but entirely unexplored. Fitting has done extensive work in establishing the existence of least fixed points in the 4-valued bilattice, and has also written about the role fixed points play in the context of logic programing. (Fitting, 1991) Varela and Kauffman's work with fixed points in "Form Dynamics" is likely to translate directly to B4.



Quehenberger, R. (2012). *Quantum Cinema, a Digital Vision: Preview.*
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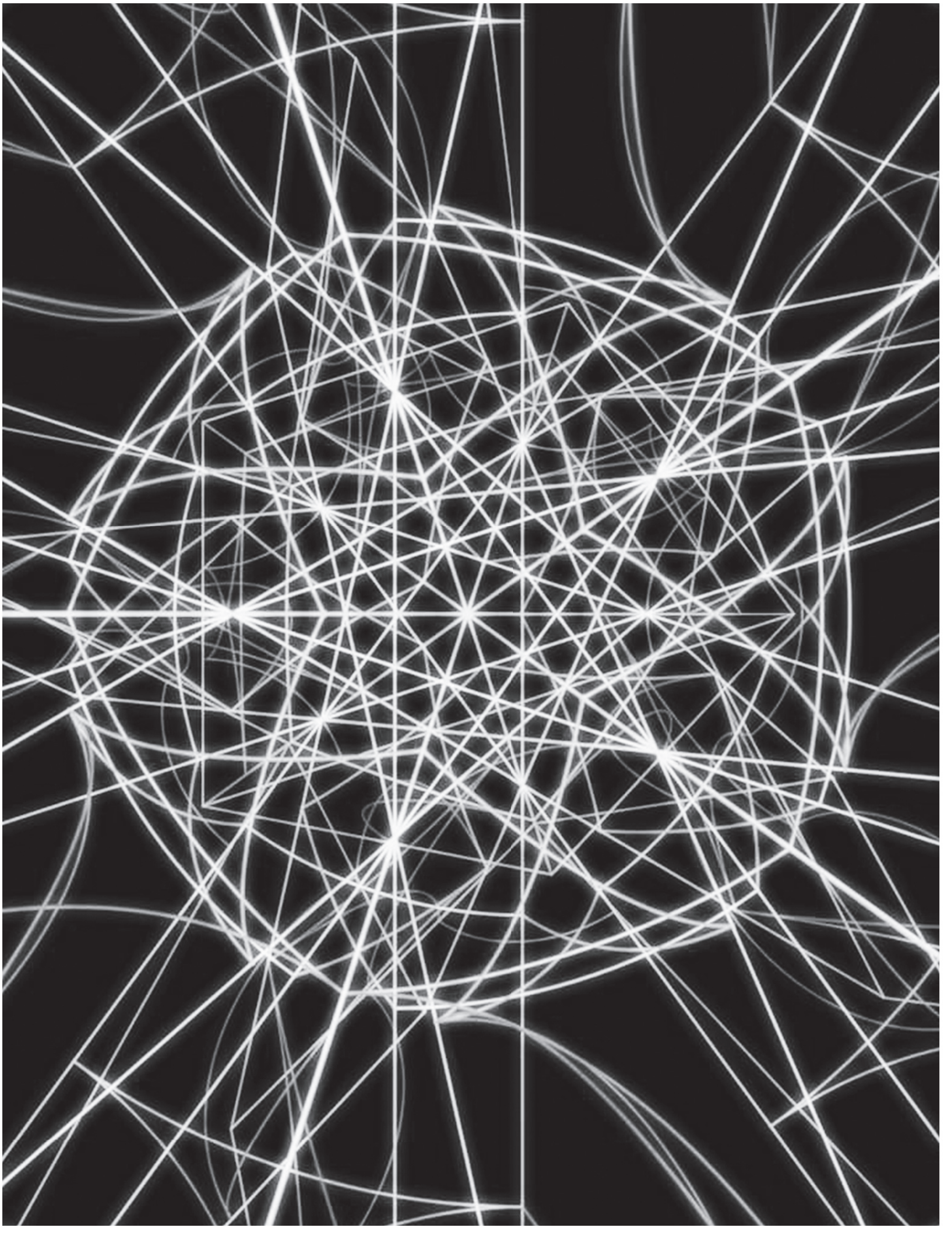

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